Supplemental Document: Noise-Resilient Imaging through Coherence Filtering

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I. DETAILED FIGURES

The image of the object is reconstructed from the cross-spectral density function by taking its inverse spatial Fourier transform. To measure the complex-valued cross-spectral density function, we use a phase-shifting wavefront inversion interferometer, a reference-free method that relies solely on intensity measurements to directly extract both the real and imaginary parts of the complex cross-spectral density function (see Methods section B in the main text for details). Accurate measurement of the real and imaginary parts requires recording interferograms precisely at phase differences of $0, \pi/2, \pi, 3\pi/2$. However, for image reconstruction, measurements can begin at any phase difference α, with the subsequent three measurements taken at $\pi/2$ phase intervals (see Methods section C in the main text for details).

Figures (S1, S2, S3) display the measured interferometric output intensity at various phase differences, specifically $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$, along with the corresponding difference intensity and the reconstructed image for the IIT Kanpur, QR code, and Grayscale wheel images under uniform noise, as presented in Fig. 4 of the main text.

Figures (S4, S5, S6) display the measured interferometric output intensity at various phase differences, specifically $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$, along with the corresponding difference intensity and the reconstructed image for QR code under spatially structured noise at various noise intensities, as presented in Fig. 5 of the main text.

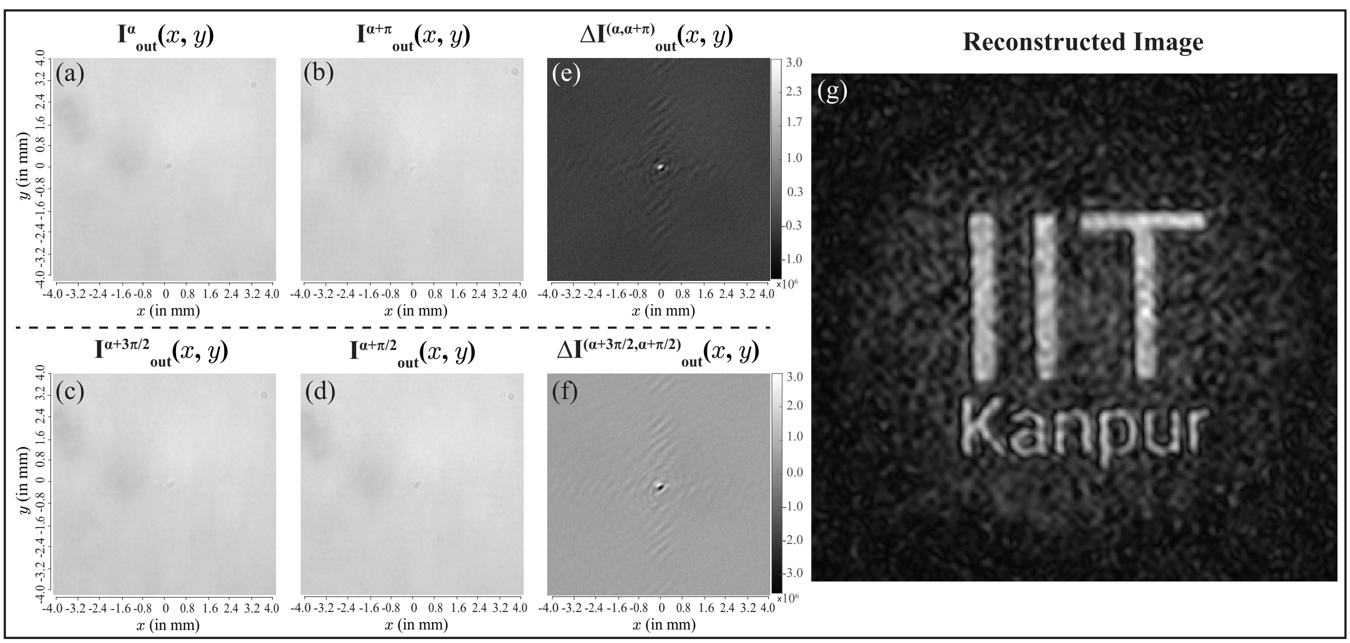

FIG. S1. Imaging plain text IIT Kanpur obscured by uniform noise with intensity 20x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\rm out}^{(0,\pi)}(x,y) + i\Delta I_{\rm out}^{(3\pi/2,\pi/2)}(x,y)$.

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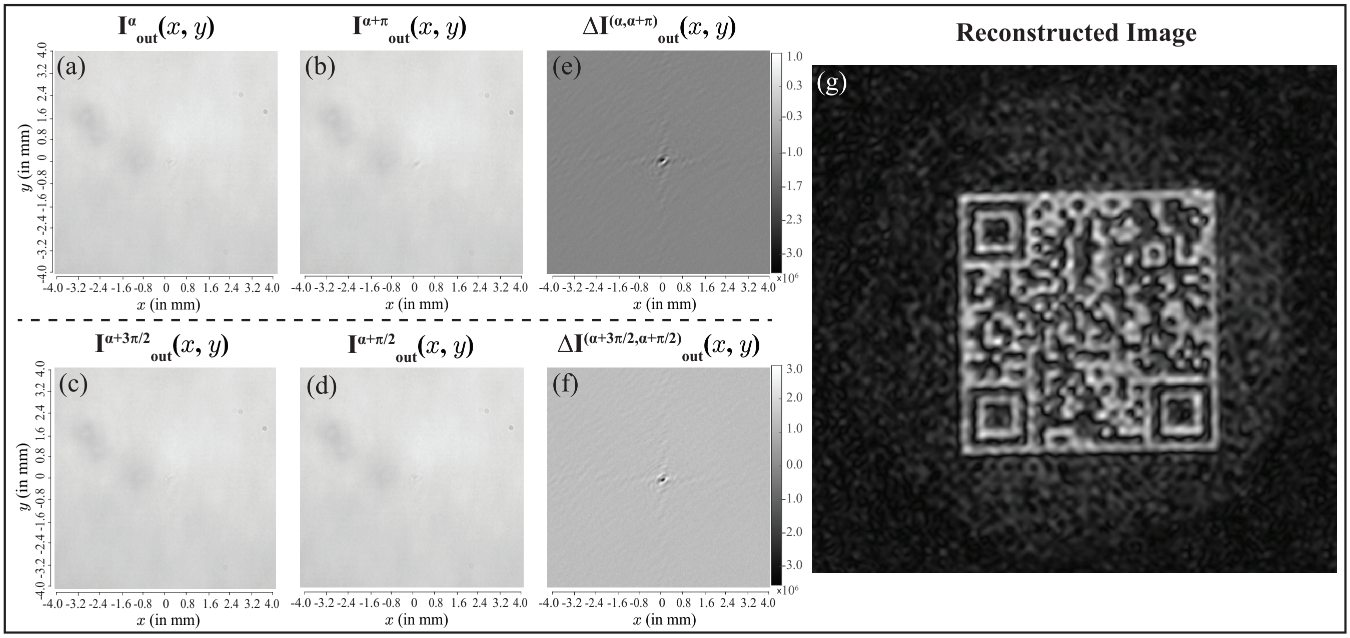

FIG. S2. Imaging QR code obscured by uniform noise with intensity 20x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\rm out}^{(0,\pi)}(x,y) + i\Delta I_{\rm out}^{(3\pi/2,\pi/2)}(x,y)$.

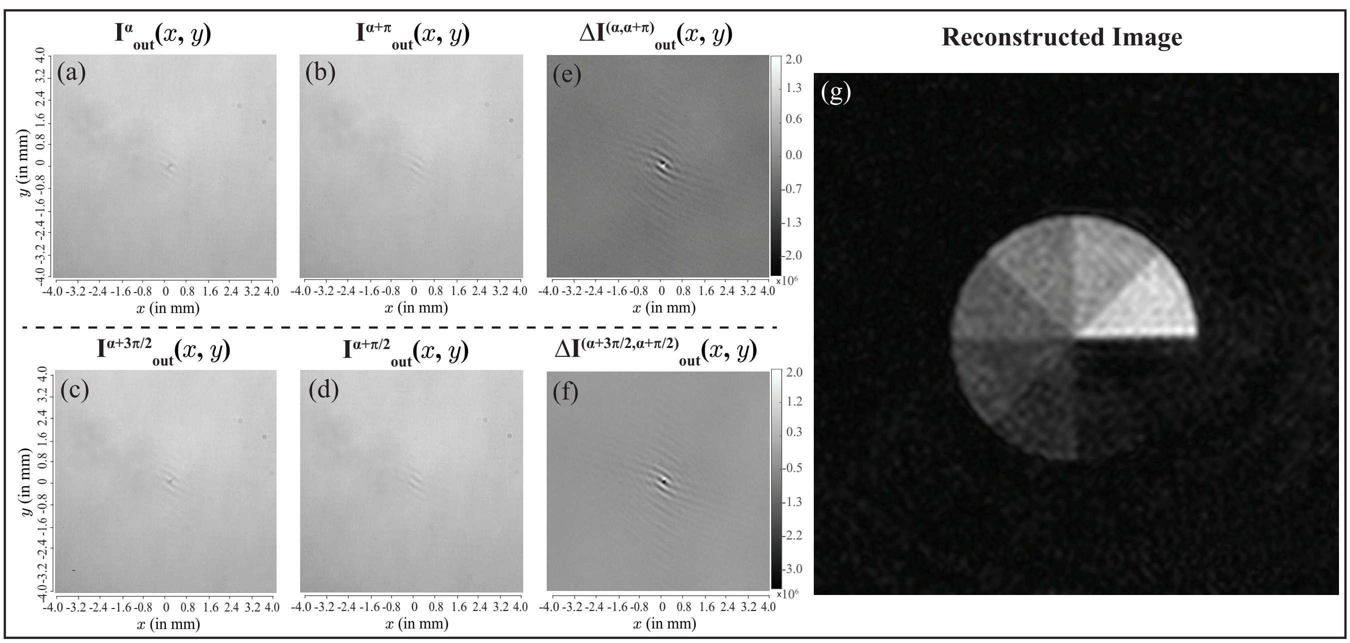

FIG. S3. Imaging Grayscale wheel obscured by uniform noise with intensity 15x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\rm out}^{(0,\pi)}(x,y) + i\Delta I_{\rm out}^{(3\pi/2,\pi/2)}(x,y)$.

II. EFFECT OF DRIFT IN PATH-LENGTH DIFFERENCE

Temporal-coherence filtering relies on setting the path-length difference Δz between the interferometer arms such that $l_c^n < \Delta z < l_c^o$; in the manuscript we set $\Delta z = m l_c^n$ with $m > 1$. The filtering efficiency is given by Eq. (13) of

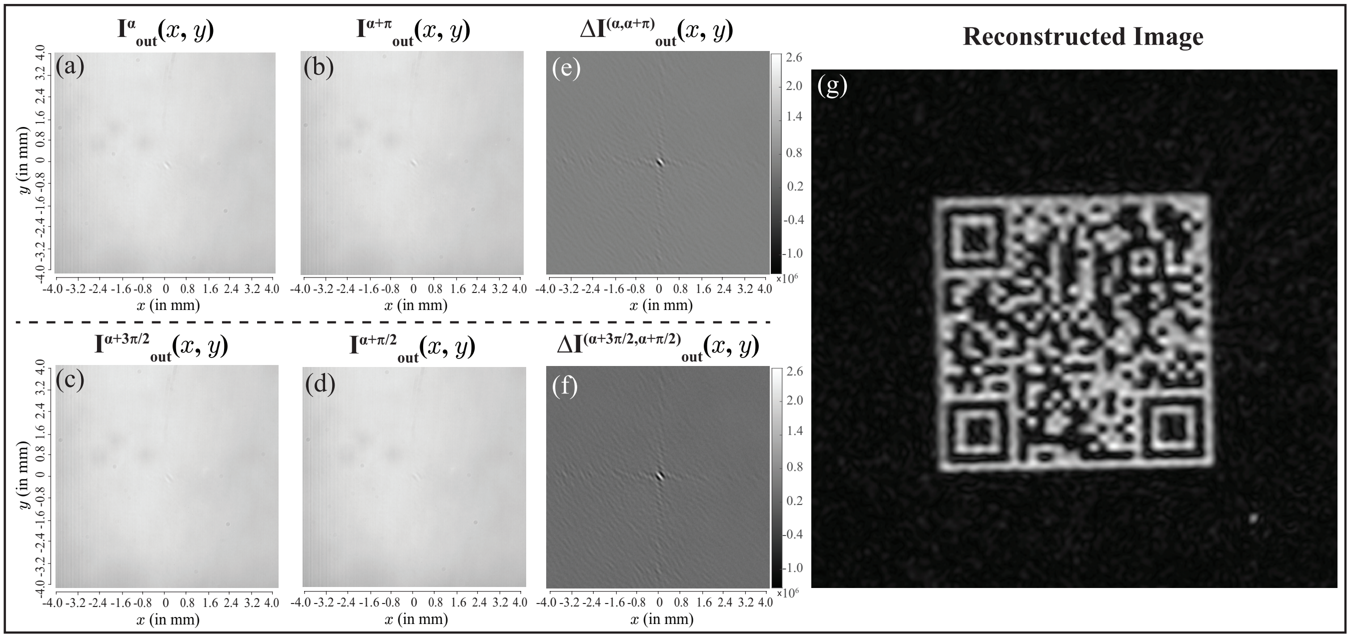

FIG. S4. Imaging QR code obscured by spatially structured noise with intensity 2x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\mathrm{out}}^{(0,\pi)}(x,y) + i\Delta I_{\mathrm{out}}^{(3\pi/2,\pi/2)}(x,y)$.

the manuscript,

$$\gamma_{\mathrm{tcf}} = \frac{A_o}{A_n} \exp\left[\frac{m^2}{2}\left(1 - \left(\frac{\Delta\omega_o}{\Delta\omega_n}\right)^2\right)\right]. \tag{1}$$

Small changes in Δz enter through $\delta m = \frac{\delta(\Delta z)}{l_c^n}$, and to first order the change in filtering efficiency is

$$\delta(\gamma_{\mathrm{tcf}}) = \gamma_{\mathrm{tcf}}\, m \left(1 - \left(\frac{\Delta\omega_o}{\Delta\omega_n}\right)^2\right) \delta m, \tag{2}$$

where $\delta(\Delta z)$ is the change in path-length difference. In the experiment, two mirrors mounted on a translation stage in one of the arms of the interferometer controls the path-length difference. Since the interferometer is vibration-isolated and enclosed, the drift due to thermal expansion and vibration is negligible. So, $\delta(\Delta z)$ is the least count of the translation stage. The least count of the translation stage used was 0.5 μm and $\delta m \approx \frac{1}{7}$.

III. EFFECT OF ERROR IN PHASE-STEPS

Phase-stepping (geometric phase) is used to extract the real and imaginary parts of the spatial cross-correlation; exact measurements at $\delta = 0, \pi/2, \pi, 3\pi/2$ recover the complex coherence directly. However, as shown in Sec. IV-C of the manuscript, if the measurement starts at an arbitrary initial phase α but subsequent steps are separated by $\pi/2$, the measured complex coherence is multiplied by a global phase factor $e^{i\alpha}$ (see Eq. (33) of the manuscript), and the reconstructed image—which uses the absolute value of the spatial inverse Fourier transform—is unaffected.

If the phase increments deviate from $\pi/2$ by amounts $\varepsilon_1, \varepsilon_2, \varepsilon_3$, then the absolute value squared of the spatial inverse Fourier transform is,

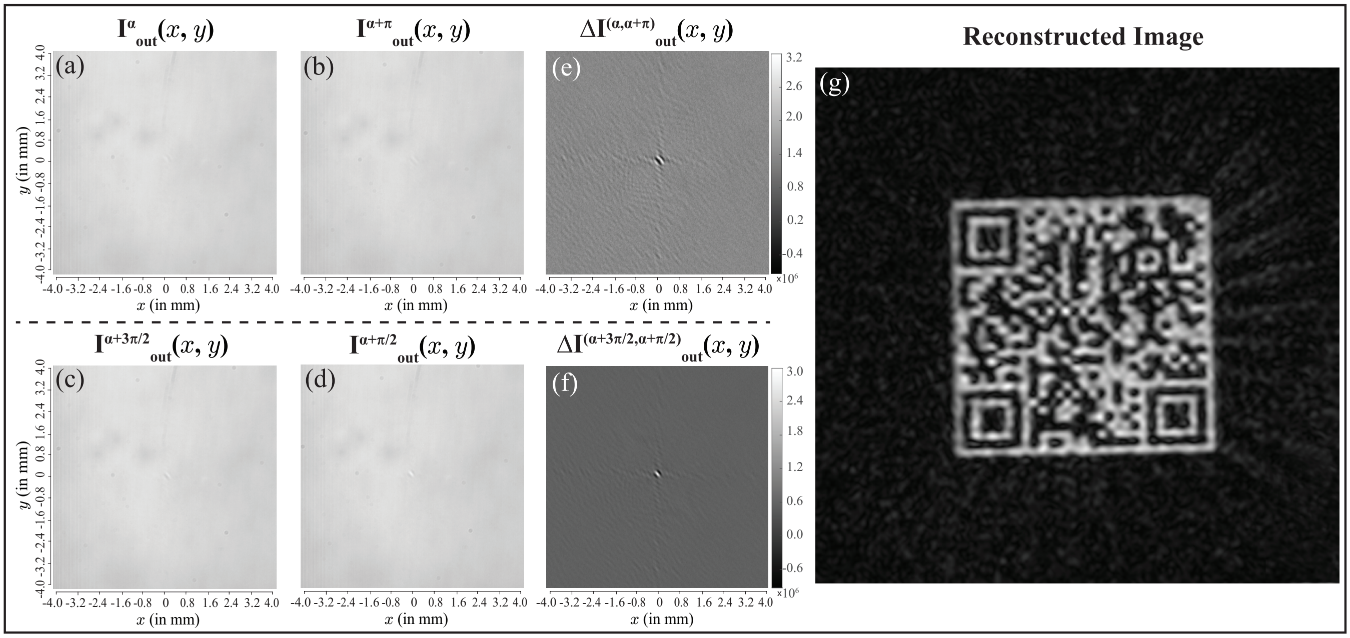

FIG. S5. Imaging QR code obscured by spatially structured noise with intensity 6x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\text{out}}^{(0,\pi)}(x,y) + i\Delta I_{\text{out}}^{(3\pi/2,\pi/2)}(x,y)$.

$$\frac{|\mathcal{F}^{-1}_{\Delta\boldsymbol{\rho}}\{\Gamma_{\mathrm{M}}(2x,-2y;\tau)\}|^2}{(4k_1k_2)^2} = \left[\left\{\frac{(1+\cos\varepsilon_1+\cos\varepsilon_2+\cos\varepsilon_3)}{2}\,I_1 + \frac{(1+\cos\varepsilon_1-\cos\varepsilon_2-\cos\varepsilon_3)}{2}\,I_2\right\}^2 + \left\{\frac{(\sin\varepsilon_1+\sin\varepsilon_2+\sin\varepsilon_3)}{2}\,I_1 + \frac{(-\sin\varepsilon_1+\sin\varepsilon_2+\sin\varepsilon_3)}{2}\,I_2\right\}^2\right]|\Omega(\tau)|^2 \quad (3)$$

where $I_1 = I_s\left(\frac{fc}{2\omega_0}q_x,\ -\frac{fc}{2\omega_0}q_y\right)$ and $I_2 = I_s\left(-\frac{fc}{2\omega_0}q_x,\ \frac{fc}{2\omega_0}q_y\right)$.

For small deviations, the above expression can be rewritten as,

$$\frac{|\mathcal{F}^{-1}_{\Delta\boldsymbol{\rho}}\{\Gamma_{\mathrm{M}}(2x,-2y;\tau)\}|}{(4k_1k_2)|\Omega(\tau)|} = \sqrt{\left\{\frac{(8-\varepsilon_1^2-\varepsilon_2^2-\varepsilon_3^2)}{4}\,I_1 + \frac{(\varepsilon_1^2-\varepsilon_2^2-\varepsilon_3^2)}{4}\,I_2\right\}^2 + \left\{\frac{(\varepsilon_1+\varepsilon_2+\varepsilon_3)}{2}\,I_1 + \frac{(-\varepsilon_1+\varepsilon_2+\varepsilon_3)}{2}\,I_2\right\}^2} \quad (4)$$

Expansion of the above equation reveals that terms linear in ε are absent.

$$|\mathcal{F}^{-1}_{\Delta\boldsymbol{\rho}}\{\Gamma_{\mathrm{M}}(2x,-2y;\tau)\}| \approx 8k_1k_2I_s\left(\frac{fc}{2\omega_0}q_x,\ -\frac{fc}{2\omega_0}q_y\right)|\Omega(\tau)| + \mathcal{O}(\varepsilon^2) \quad (5)$$

Thus, to the first order, there is no impact on image formation.

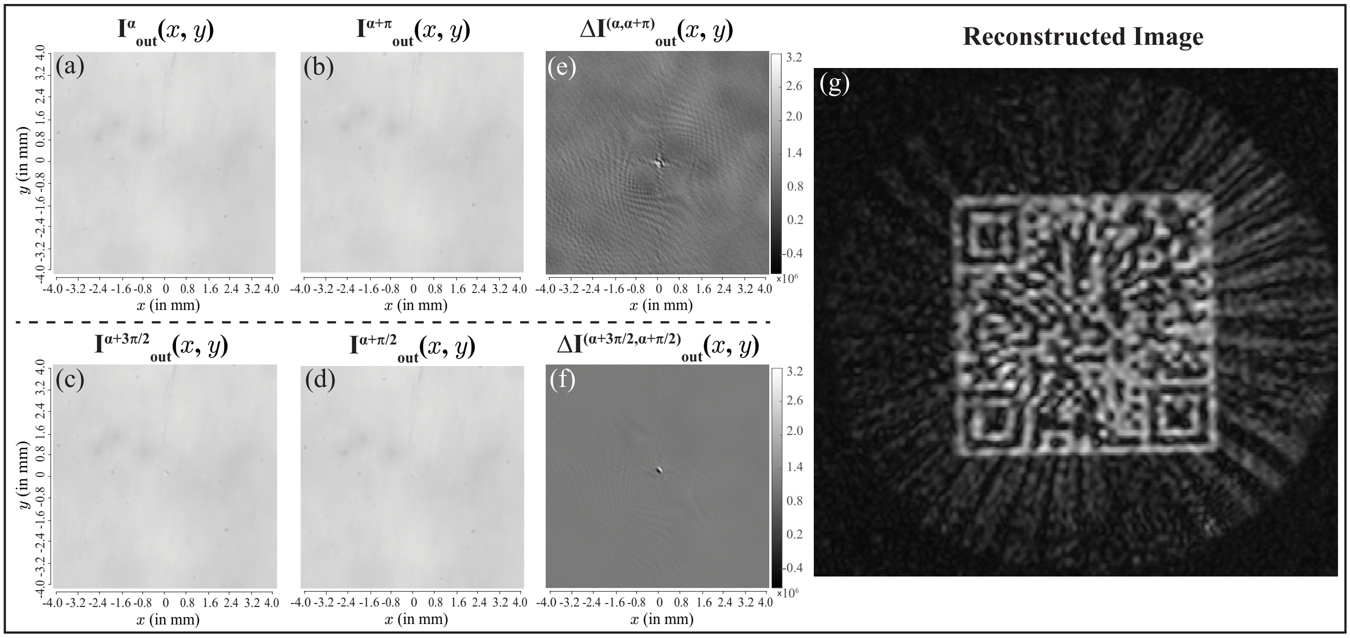

FIG. S6. QR code obscured by spatially structured noise with intensity 20x the object field's intensity: (a)-(d) The measured interferometric output intensity at different phase differences $\delta = \alpha, \alpha+\pi, \alpha+\pi/2, \alpha+3\pi/2$. (e)-(f) The difference intensity (g) The reconstructed image by taking the inverse Fourier transform of the difference intensity $\Delta I_{\rm out}^{(0,\pi)}(x,y) + i\Delta I_{\rm out}^{(3\pi/2,\pi/2)}(x,y)$.